CFTP/10-001

Renormalization-group constraints on Yukawa alignment in multi-Higgs-doublet models

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14 January 2010

Abstract

We write down the renormalization-group equations for the Yukawa-coupling matrices in a general multi-Higgs-doublet model. We then assume that the matrices of the Yukawa couplings of the various Higgs doublets to right-handed fermions of fixed quantum numbers are all proportional to each other. We demonstrate that, in the case of the two-Higgs-doublet model, this proportionality is preserved by the renormalization-group running only in the cases of the standard type-I, II, X, and Y models. We furthermore show that a similar result holds even when there are more than two Higgs doublets: the Yukawa-coupling matrices to fermions of a given electric charge remain proportional under the renormalization-group running if and only if there is a basis for the Higgs doublets in which all the fermions of a given electric charge couple to only one Higgs doublet.

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The standard model of the electroweak interactions has an extension with two Higgs doublets, called the two-Higgs-doublet model (THDM). In that extension the Yukawa couplings are given by

$$\mathcal{L}_{\mathrm{Y}} = -\bar{Q}_L \left[\left(\Gamma_1 \phi_1 + \Gamma_2 \phi_2 \right) n_R + \left(\Delta_1 \tilde{\phi}_1 + \Delta_2 \tilde{\phi}_2 \right) p_R \right] - \bar{L}_L \left(\Pi_1 \phi_1 + \Pi_2 \phi_2 \right) \ell_R + \mathrm{H.c.}, \quad (1)$$

where the ϕ_k ($k = 1, 2$) are the Higgs doublets, $\tilde{\phi}_k \equiv i \tau_2 \phi_k^*$, and Q_L, L_L, n_R, p_R, and ℓ_R are 3-vectors in flavour space; Γ_k, Δ_k, and Π_k are the 3×3 complex matrices of the Yukawa couplings to the right-handed down-type quarks, up-type quarks, and charged leptons, respectively.

Recently [1], it has been proposed that the Yukawa-coupling matrices may be proportional to each other:

$$\begin{aligned} \Gamma_2 &= d \Gamma_1, & (2a) \\ \Delta_2 &= u \Delta_1, & (2b) \\ \Pi_2 &= e \Pi_1, & (2c) \end{aligned}$$

where d, u, and e are complex numbers. This 'Yukawa alignment' might be the result of some symmetry and, as such, be stable under renormalization. The purpose of this paper is to investigate that possibility by making use of the renormalization-group equations (RGE).

The one-loop RGE for a model with n Higgs doublets have been presented in [2] for the special case in which $\mathcal{L}_{\mathrm{Y}}$ only includes the matrices Π_k. It is easy to extend them to the most general case. Let μ be the renormalization scale and $\mathcal{D} \equiv 16 \pi^2 \mu \left(\mathrm{d} / \mathrm{d} \mu \right)$. Then,

$$\begin{aligned} \mathcal{D} \Gamma_k &= a_\Gamma \Gamma_k & (3a) \\ &+ \sum_{l=1}^{n} \left[3 \, \mathrm{tr} \left(\Gamma_k \Gamma_l^\dagger + \Delta_k^\dagger \Delta_l \right) + \mathrm{tr} \left(\Pi_k \Pi_l^\dagger \right) \right] \Gamma_l & (3b) \\ &+ \sum_{l=1}^{n} \left(-2 \, \Delta_l \Delta_k^\dagger \Gamma_l + \Gamma_k \Gamma_l^\dagger \Gamma_l + \frac{1}{2} \, \Delta_l \Delta_l^\dagger \Gamma_k + \frac{1}{2} \, \Gamma_l \Gamma_l^\dagger \Gamma_k \right), & (3c) \end{aligned}$$

$$\begin{aligned} \mathcal{D} \Delta_k &= a_\Delta \Delta_k & (4a) \\ &+ \sum_{l=1}^{n} \left[3 \, \mathrm{tr} \left(\Delta_k \Delta_l^\dagger + \Gamma_k^\dagger \Gamma_l \right) + \mathrm{tr} \left(\Pi_k^\dagger \Pi_l \right) \right] \Delta_l & (4b) \\ &+ \sum_{l=1}^{n} \left(-2 \, \Gamma_l \Gamma_k^\dagger \Delta_l + \Delta_k \Delta_l^\dagger \Delta_l + \frac{1}{2} \, \Gamma_l \Gamma_l^\dagger \Delta_k + \frac{1}{2} \, \Delta_l \Delta_l^\dagger \Delta_k \right), & (4c) \end{aligned}$$

$$\begin{aligned} \mathcal{D} \Pi_k &= a_\Pi \Pi_k & (5a) \\ &+ \sum_{l=1}^{n} \left[3 \, \mathrm{tr} \left(\Gamma_k \Gamma_l^\dagger + \Delta_k^\dagger \Delta_l \right) + \mathrm{tr} \left(\Pi_k \Pi_l^\dagger \right) \right] \Pi_l & (5b) \\ &+ \sum_{l=1}^{n} \left(\Pi_k \Pi_l^\dagger \Pi_l + \frac{1}{2} \, \Pi_l \Pi_l^\dagger \Pi_k \right), & (5c) \end{aligned}$$

where tr denotes the trace and the factors 3 in (3b), (4b), and (5b) are from colour. Furthermore,

$$\begin{aligned} a_\Gamma &= -8g_s^2 - \frac{9}{4}\, g^2 - \frac{5}{12}\, g'^2, && (6a)\\ a_\Delta &= -8g_s^2 - \frac{9}{4}\, g^2 - \frac{17}{12}\, g'^2, && (6b)\\ a_\Pi &= -\frac{9}{4}\, g^2 - \frac{15}{4}\, g'^2, && (6c) \end{aligned}$$

where g_s, g, and g' are the gauge coupling constants of $SU(3)_{\mathrm{colour}}$, $SU(2)$, and $U(1)$, respectively.[1] We emphasize that equations (3)–(6) are valid for a model with an *arbitrary* number of Higgs doublets and for any number of fermion generations.

Let us examine the contribution (3b) in the THDM under the alignment hypothesis (2). One obtains

$$\begin{aligned} \mathcal{D}\Gamma_1 &= \Big[\left(3 + 3\,|d|^2\right) \mathrm{tr}\left(\Gamma_1\Gamma_1^\dagger\right) + \left(3 + 3ud\right) \mathrm{tr}\left(\Delta_1^\dagger\Delta_1\right) \\ &\quad + \left(1 + e^* d\right) \mathrm{tr}\left(\Pi_1\Pi_1^\dagger\right)\Big]\,\Gamma_1 + \cdots\,, && (7a)\\ \mathcal{D}\Gamma_2 &= \Big[\left(3\,|d|^2 d + 3d\right) \mathrm{tr}\left(\Gamma_1\Gamma_1^\dagger\right) + \left(3\,|u|^2 d + 3u^*\right) \mathrm{tr}\left(\Delta_1^\dagger\Delta_1\right) \\ &\quad + \left(|e|^2 d + e\right) \mathrm{tr}\left(\Pi_1\Pi_1^\dagger\right)\Big]\,\Gamma_1 + \cdots\,. && (7b) \end{aligned}$$

We now require that the hypothesis (2) be stable under the RGE, *i.e.* we require

$$\begin{aligned} \mathcal{D}\Gamma_2 &= d\left(\mathcal{D}\Gamma_1\right), && (8a)\\ \mathcal{D}\Delta_2 &= u\left(\mathcal{D}\Delta_1\right), && (8b)\\ \mathcal{D}\Pi_2 &= e\left(\mathcal{D}\Pi_1\right). && (8c) \end{aligned}$$

Using (7), one finds that (8a) leads to

$$\begin{aligned} \left(u^* - d\right)\left(1 + ud\right) &= 0, && (9a)\\ \left(e - d\right)\left(1 + e^* d\right) &= 0. && (9b) \end{aligned}$$

It is easy to check that the condition (9a) ensures that (8a) also holds for the terms in (3c). One may furthremore check that (8b) and (8c) do not yield any further conditions on d, u, and e beyond (9). We have thus found that the alignment hypothesis is (one-loop) renormalization-group invariant only in one of the four cases

$$d = e = u^*, \tag{10a}$$

$$d = e = -\frac{1}{u}, \tag{10b}$$

$$d^* = u = -\frac{1}{e}, \tag{10c}$$

$$u = e^* = -\frac{1}{d}. \tag{10d}$$

[1] The normalization of g' is such that the ϕ_k have hypercharge 1/2.

Instead of using the basis $(\phi_1, \phi_2)^T$ for the two Higgs doublets, one may use any other basis, related to that one through an $SU(2)$ transformation. In particular, one may use any of the following three bases:

$$\begin{pmatrix} \phi_{d1} \\ \phi_{d2} \end{pmatrix} = \left(1 + |d|^2\right)^{-1/2} \begin{pmatrix} 1 & d \\ -d^* & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{11a}$$

$$\begin{pmatrix} \phi_{u1} \\ \phi_{u2} \end{pmatrix} = \left(1 + |u|^2\right)^{-1/2} \begin{pmatrix} 1 & u^* \\ -u & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{11b}$$

$$\begin{pmatrix} \phi_{e1} \\ \phi_{e2} \end{pmatrix} = \left(1 + |e|^2\right)^{-1/2} \begin{pmatrix} 1 & e \\ -e^* & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{11c}$$

If the alignment conditions (2) hold, then the Yukawa Lagrangian (1) may be rewritten

$$\mathcal{L}_\mathrm{Y} = -\bar{Q}_L \left(\phi_{d1} \sqrt{1 + |d|^2}\, \Gamma_1 n_R + \tilde{\phi}_{u1} \sqrt{1 + |u|^2}\, \Delta_1 p_R \right) - \bar{L}_L \phi_{e1} \sqrt{1 + |e|^2}\, \Pi_1 \ell_R + \mathrm{H.c.}, \tag{12}$$

where $\tilde{\phi}_{u1} \equiv i \tau_2 \phi_{u1}^*$. The various cases (10) may then be interpreted in the following way:

- In the case (10a), $\phi_{d1} = \phi_{u1} = \phi_{e1}$, hence

$$\frac{\mathcal{L}_\mathrm{Y}}{\sqrt{1 + |d|^2}} = -\bar{Q}_L \left(\phi_{d1} \Gamma_1 n_R + \tilde{\phi}_{d1} \Delta_1 p_R \right) - \bar{L}_L \phi_{d1} \Pi_1 \ell_R + \mathrm{H.c.},$$

  which means that, in the basis (ϕ_{d1}, ϕ_{d2}), only ϕ_{d1} has Yukawa couplings. This is called the type-I THDM [3].

- In the case (10b),

$$\frac{\mathcal{L}_\mathrm{Y}}{\sqrt{1 + |d|^2}} = -\bar{Q}_L \left(\phi_{d1} \Gamma_1 n_R - \tilde{\phi}_{d2} \frac{\Delta_1}{d}\, p_R \right) - \bar{L}_L \phi_{d1} \Pi_1 \ell_R + \mathrm{H.c.},$$

  which means that only ϕ_{d1} has Yukawa couplings to the down-type quarks and to the charged leptons, while only ϕ_{d2} has Yukawa couplings to the up-type quarks. This is called the type-II THDM [4].

- In the case (10c),

$$\frac{\mathcal{L}_\mathrm{Y}}{\sqrt{1 + |d|^2}} = -\bar{Q}_L \left(\phi_{d1} \Gamma_1 n_R + \tilde{\phi}_{d1} \Delta_1 p_R \right) + \bar{L}_L \phi_{d2} \frac{\Pi_1}{d^*}\, \ell_R + \mathrm{H.c.},$$

  which means that only ϕ_{d1} has Yukawa couplings to the quarks while only ϕ_{d2} has Yukawa couplings to the charged leptons. This is called the type-X THDM [5].

- In the case (10d)

$$\frac{\mathcal{L}_\mathrm{Y}}{\sqrt{1 + |d|^2}} = -\bar{Q}_L \left(\phi_{d1} \Gamma_1 n_R - \tilde{\phi}_{d2} \frac{\Delta_1}{d}\, p_R \right) + \bar{L}_L \phi_{d2} \frac{\Pi_1}{d^*}\, \ell_R + \mathrm{H.c.},$$

which means that only ϕ_{d1} has Yukawa couplings to the down-type quarks while only ϕ_{d2} has Yukawa couplings to the charged leptons and to the up-type quarks. This is called the type-Y THDM [5].

It is well known that all these cases may be obtained through the imposition of appropriate $\mathbb{Z}_2$ symmetries on the Yukawa Lagrangian (1).

We proceed to analyze the case of an arbitrary number n of Higgs doublets, where

$$\mathcal{L}_{\rm Y} = -\bar{Q}_L \sum_{k=1}^{n} \left(\Gamma_k \phi_k n_R + \Delta_k \tilde{\phi}_k p_R \right) - \bar{L}_L \sum_{k=1}^{n} \Pi_k \phi_k \ell_R + \text{H.c.}, \tag{13}$$

with a generalized alignment hypothesis

$$\begin{aligned} \Gamma_k &= d_k \Gamma_1, & \text{(14a)} \\ \Delta_k &= u_k \Delta_1, & \text{(14b)} \\ \Pi_k &= e_k \Pi_1, & \text{(14c)} \end{aligned}$$

where $d_1 = u_1 = e_1 = 1$. Inserting the conditions (14) in equation (3) leads to

$$\begin{aligned} \mathcal{D}\Gamma_k &= a_\Gamma\, d_k\, \Gamma_1 \\ &+ \sum_{l=1}^{n} \left[3 d_k \left| d_l \right|^2 \operatorname{tr} \left(\Gamma_1 \Gamma_1^\dagger \right) + 3 u_k^* u_l d_l \operatorname{tr} \left(\Delta_1^\dagger \Delta_1 \right) + e_k e_l^* d_l \operatorname{tr} \left(\Pi_1 \Pi_1^\dagger \right) \right] \Gamma_1 \\ &+ \sum_{l=1}^{n} \left[\left(-2 u_l u_k^* d_l + \frac{1}{2} \left| u_l \right|^2 d_k \right) \Delta_1 \Delta_1^\dagger \Gamma_1 + \frac{3}{2} |d_l|^2 d_k \Gamma_1 \Gamma_1^\dagger \Gamma_1 \right]. \end{aligned} \tag{15}$$

It follows that

$$\begin{aligned} \mathcal{D}\Gamma_k - d_k \mathcal{D}\Gamma_1 &= \sum_{l=1}^{n} \left[3 \left(u_k^* - d_k \right) u_l d_l \operatorname{tr} \left(\Delta_1^\dagger \Delta_1 \right) + \left(e_k - d_k \right) e_l^* d_l \operatorname{tr} \left(\Pi_1 \Pi_1^\dagger \right) \right] \Gamma_1 \\ & \quad -2 \left(u_k^* - d_k \right) \sum_{l=1}^{n} u_l d_l \Delta_1 \Delta_1^\dagger \Gamma_1. \end{aligned} \tag{16}$$

We require the quantity (16) to vanish for all k. A similar requirement is imposed on $\mathcal{D}\Delta_k - u_k \mathcal{D}\Delta_1$ and on $\mathcal{D}\Pi_k - e_k \mathcal{D}\Pi_1$. We find that the generalized alignment hypothesis (14) is preserved by the renormalization-group evolution if and only if

$$\begin{aligned} \left(u_k^* - d_k \right) \sum_{l=1}^{n} u_l d_l &= 0, & \text{(17a)} \\ \left(e_k - d_k \right) \sum_{l=1}^{n} e_l^* d_l &= 0, & \text{(17b)} \\ \left(u_k^* - e_k \right) \sum_{l=1}^{n} u_l e_l &= 0 & \text{(17c)} \end{aligned}$$

are satisfied for all k. Now, if the generalized alignment hypothesis holds, then the Yukawa Lagrangian (13) may be rewritten as

$$\mathcal{L}_{\rm Y} = -\bar{Q}_L \left(N_d \phi_{d1} \Gamma_1 n_R + N_u \tilde{\phi}_{u1} \Delta_1 p_R \right) - \bar{L}_L N_e \phi_{e1} \Pi_1 \ell_R + \text{H.c.}, \tag{18}$$

where

$$N_d \phi_{d1} = \sum_{k=1}^{n} d_k \phi_k, \qquad N_d = \sqrt{\sum_{k=1}^{n} \left| d_k \right|^2}, \tag{19a}$$

$$N_u \phi_{u1} = \sum_{k=1}^{n} u_k^* \phi_k, \qquad N_u = \sqrt{\sum_{k=1}^{n} \left| u_k \right|^2}, \tag{19b}$$

$$N_e \phi_{e1} = \sum_{k=1}^{n} e_k \phi_k, \qquad N_e = \sqrt{\sum_{k=1}^{n} \left| e_k \right|^2}. \tag{19c}$$

Thus, condition (17a) means that either $\phi_{u1} = \phi_{d1}$ or else ϕ_{u1} and ϕ_{d1} are orthogonal; the conditions (17b) and (17c) have analogous interpretations. This finally leaves us with only five possibilities:

1. $\phi_{d1} = \phi_{u1} = \phi_{e1}$;
2. ϕ_{u1} is orthogonal to $\phi_{d1} = \phi_{e1}$;
3. ϕ_{e1} is orthogonal to $\phi_{d1} = \phi_{u1}$;
4. ϕ_{d1} is orthogonal to $\phi_{e1} = \phi_{u1}$;
5. ϕ_{d1}, ϕ_{u1}, and ϕ_{e1} are all orthogonal to each other.

The possibilities 1–4 reproduce the four cases (10a)–(10d) already present in the THDM. The possibility 5 is new: if there are three or more Higgs doublets, it is possible that each charged-fermion sector couples to a separate Higgs doublet—this is of course impossible in the THDM.

It is easy to show that all five cases 1–5 may be enforced through simple $\mathbb{Z}_2$ symmetries.

To summarize, in this paper we have shown that Yukawa alignment in the two-Higgs-doublet model, as proposed in [1], can only be stable under the renormalization group in the well-known cases in which only one Higgs doublet has Yukawa couplings to the right-handed fermions of each electric charge. All other cases of Yukawa alignment are not consistent with the renormalization group, which means that, either they cannot be enforced by any symmetries, or they can be enforced by symmetries only in the context of a model larger than the THDM. We have generalized the notion of alignment to models with an arbitrary number of Higgs doublets and have shown that, also in that generalization, alignment is only stable under the RGE when the right-handed fermions of each electric charge all couple to only one Higgs doublet.

Acknowledgements: We thank A. Pich and P. Tuzón for reading and commenting on the manuscript. The work of P.M.F. is supported in part by the Portuguese *Fundação para a Ciência e a Tecnologia* (FCT) under contract PTDC/FIS/70156/2006. The work of L.L. and of J.P.S. is funded by FCT through the project U777–Plurianual.

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